PATTERNED ARMOR PERFORMANCE AGAINST MULTIPLE IMPACTS

William S. de Rosset
U.S. Army Research Laboratory
APG, MD 21005

LTC Rodney Sturdivant
United States Military Academy
West Point, NY 10966

ABSTRACT

Two approaches have been taken to model the effect of multiple impacts on patterned armor. The first is a simple analytical model that assumes a uniform random distribution of impacts. The second is a more sophisticated computer model that allows for a Gaussian distribution of impacts around the aim point. The analytic model is used to show how areas of weakness in patterned armor can affect performance. In addition, the analytic model can be exercised to conduct quick trade-off studies between patterned armor concepts. The computer model, developed specifically to determine the effects of a small number of impacts, is used to calculate the effects of dispersion of impact points and aim point.

1. INTRODUCTION

During the period from World War II to the first Persian Gulf War armor research and development efforts emphasized protecting heavy fighting vehicles against single impacts of kinetic energy penetrators or shaped charge warheads fired over long open ranges. More recently, the emphasis has been on the protection of lightly armored vehicles against small arms, Improvised Explosive Devices (IEDs), and rocket-propelled grenades (RPGs) over short ranges in an urban environment. This radical shift in emphasis has prompted a new approach to evaluating armor performance, especially if the light armor in question is a patterned armor and there are multiple impacts on the armor due to automatic weapons fire or fragmenting warheads.

Patterned armor consists of repeating individual cells that are arrayed on a vehicle so as to provide the best coverage. While the cells are highly effective in stopping a single impact, in many instances they are incapable of defeating a threat that strikes the same cell twice. In addition, patterned armors tend to have lower ballistic performance at the edges of the cells. Examples of patterned armor include reactive armor boxes, ceramic tiles, and P900 armor.

Two approaches were taken to provide estimates of patterned armor performance against multiple impacts produced by automatic weapons fire. First, a simple analytic model was developed based on probability theory that estimates overall armor performance for a variety of situations. Input quantities that can be varied to conduct sensitivity studies include cell size, ballistic performance of individual cells, and impact dispersion. The output is the probability that the patterned armor can defeat a certain number of rounds fired from a given range. Assumptions can also be changed concerning the ability of a single cell to defeat more than one impact. We show that, for a specific performance specification, it is more advantageous to design a patterned armor whose cells are capable of stopping more than one impact as compared to a patterned armor whose cells have higher performance against individual attacks but can defeat only one impact. This simple model can also be used to examine the effect of poor ballistic performance near the edges of the cells.

The analytic model is useful when the impact dispersion is large or the cell size is small. Certain approximations used in the model break down when there are only a few impacts falling on a small number of cells. For this reason, we developed a computer model that involves more sophisticated Monte Carlo methods. The model is coded in Mathematica®, a computer software package readily available to researchers at both the U.S. Army Research Laboratory and the United States Military Academy. Certain sensitivity studies were repeated with an emphasis on small numbers of impacts over a small area (e.g., short range attack). The

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE	2. REPORT TYPE	3. DATES COVERED
01 NOV 2006	**N/A**	**-**

4. TITLE AND SUBTITLE
Patterned Armor Performance Against Multiple Impacts

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
U.S. Army Research Laboratory APG, MD 21005

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release, distribution unlimited

13. SUPPLEMENTARY NOTES
See also ADM002075., The original document contains color images.

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			
unclassified	**unclassified**	**unclassified**	**UU**	**7**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

computer model was also used to show the effect of hit location on an array of cells.

2. ANALYTIC MODEL

Both the analytic model and computer model are based on the following assumptions. First, the armor consists of repeating cells that cover the critical areas of the armored vehicle. For simplicity, the shape of these cells is taken to be square, although rectangles and hexagons could also be used. Second, it is assumed that a cell can defeat the first impact of the threat round. This assumption can be relaxed to allow a zone of weakness near the edge of the cell. However, the cell cannot defeat a second impact (this restriction can also be relaxed). Finally, a cell adjacent to another cell that has been impacted can defeat the threat round. This type of performance is referred to as "multiple-hit" capability.

The threat examined for the analytic model is further restricted to automatic weapons fire. It is assumed that hits from the weapon fall randomly within an area *A*. It is assumed that the number of cells within *A* is large so that the effect of hitting a cell not totally contained within *A* is small. The impacting bullet is assumed to have negligible lateral dimensions so that only one cell is impacted at a time.

The probability that the armor array defeats the first round is 1. However, an area D^2 is now left exposed, where *D* is the edge length of the cell. The probability that the second round is also defeated, P_2, is then given by

$$P_2 = 1 - D^2 / A. \tag{1}$$

As more cells are removed by subsequent impacts, the probability that all impacts up to and including the *n*th one is defeated is given by

$$P_n = (1 - D^2/A)(1 - 2D^2/A... \\ (1-(n-1)D^2/A \tag{2}$$

If we approximate *A* by ND^2, where *N* is the number of cells contained within the impact area, then equation (2) reduces to

$$P_n = N!/(N^n(N-n)!) \tag{3}$$

The model can be extended to the case where there exists an area of weakness in a cell. This area of weakness might result from the support or containment system of the cell. For instance, if the cell cannot defeat the threat anywhere within a distance δ from each edge, then the probability that the first round is defeated is given by

$$P_1 = (1 - 2\delta / D)^2. \tag{4}$$

In a similar manner used to develop equation 3, the expression for P_n is given in this case by

$$P_n = \{N!/(N^n(N-n)!)\}(1-2\delta/D)^n. \tag{5}$$

The performance of two patterned armor arrays can be compared with this model. Assume that both are identical except that there is an area of individual cell weakness in the second array that extends 2 mm from the cell edge. Let *D* = 100mm and *A* = 2 square meters. The performance of each armor as a function of *n* is shown in Figure 1. The model calculations emphasize the importance of eliminating vulnerable areas of a cell.

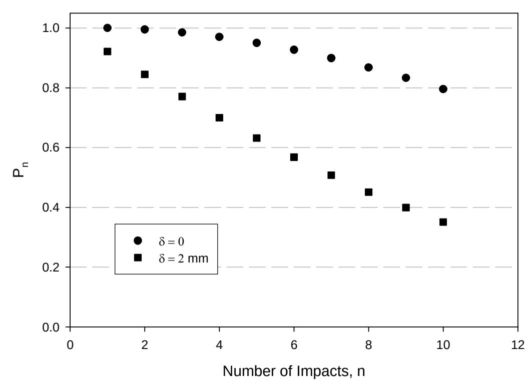

Figure 1. Performance comparison between armors with and without a vulnerable area

The model can be expanded to include the effects of distance from the weapon to the target, normally referred to as range. In general, the dispersion of a weapon and ammunition system is defined as the standard deviation of the miss distances divided by the target-to-weapon distance (range). The dispersion is expressed in milli-radians (mrad). The average miss distance, μ, is given by

$$\mu = R * \sigma, \tag{6}$$

where R is the range and σ is the weapon dispersion. It can be shown (de Rosset and Wald, 2002) that

$$A(R) = 8.71\mu^2 . \qquad (7)$$

In addition, the velocity of a projectile decreases with increasing range due to air drag. Armors are usually designed to defeat a projectile at a certain velocity and below. If that velocity is the muzzle velocity of the weapon, then the range will have no effect as far as single-impact performance is concerned. However, if the armor can defeat the projectile at a velocity less than muzzle velocity, range will make a difference.

As an example of how range might affect performance, consider a specific patterned armor that has the following characteristics. First, the armor cell can defeat the projectile at a velocity of 850m/s or lower over most of its area. There is a small area near the cell edge that cannot stop the projectile until the projectile velocity is 700 m/s or below. Let δ = 2mm and D = 100mm. For the projectile, assume that its velocity V as a function of range is given by

$$V = 900 - 0.4R, \quad 0 < R < 1000, \qquad (8)$$

where V is in meters per second and R is in meters. Take σ as 1 mrad. These weapon and target characteristics result in a value of P_7 as shown in Figure 2. The apparent discontinuous jump in P_7 occurs as a result of assuming a single velocity that divides cell defeat from non-defeat. In reality, there is likely to be a gradual transition of target performance over a range of velocities.

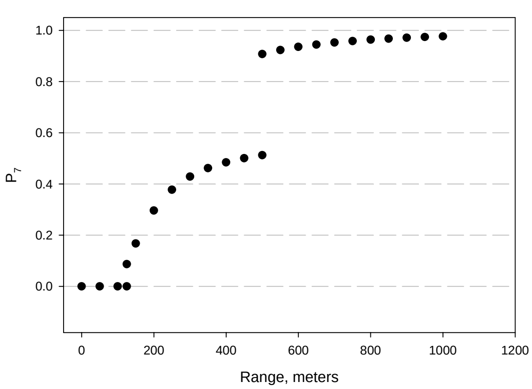

Figure 2. P_7 as a function of range

From 0 to 125 meters, the cells cannot defeat the projectile, so that P_7 is zero. At 125 meters range, the major portion of each cell is now able to defeat the projectile whose velocity has fallen below 850 m/s. The increase in P_7 from 125 to 500 meters range is due to the increase in the area A. At 500 meters range the entire cell is capable of defeating the projectile.

The analytic model can be used to guide the design of patterned armor. For instance, the model shows the armor performance increasing with decreasing cell lateral dimensions. However, depending on what armor technology is being used, there is a minimum size cell that can be used and still achieve multiple-hit capability.

The model suggests that in some cases P_n can be held constant while varying both D and δ. For instance, the value of P_2 for D = 100 mm and δ = 0 is 0.995. The same vale of P_2 can be obtained with the values of D and δ shown in Table 1. The increase in performance due to a smaller cell size is offset by the larger value of δ. The values of δ calculated for this case are too small to be of much use to the armor designer, and it is expected that they would decrease as n becomes larger.

Table 1. Values of D and δ for Constant P_2

D (mm)	δ (mm)
100.0	0
95.5	0.01
90.3	0.02
83.8	0.03
74.5	0.04

The model also allows the value of unusual armor concepts to be evaluated before extensive research is conducted. Suppose that a cell in a patterned armor had the capability of withstanding two impacts at reasonably high velocity. (There will be some impact velocity, albeit low, where a ceramic armor tile can defeat two impacts.) Call this a two-impact armor. The usual rules governing probability can be applied to determine the likelihood the armor will defeat the first n shots. Call this likelihood $P(n)$ to distinguish it from that associated with one-impact cells. By definition, $P(1) = P(2) = 1$. It can be shown (de Rosset, 2005) that

$$P(3) = 1 - 1/N^2 . \qquad (9)$$

Explicit formulas for $P(n)$ have been generated for values of n up to and including 7. The expressions become increasingly complicated, and there is no equivalent to the simple forms given for P values as shown in equations 3 and 5 for any value of n. For instance, the expression for $P(7)$ is

$$P(7) = 1 - 35/N^2 + 105/N^3 - 56/N^4 - 105/N^5 + 90/N^6 . \quad (10)$$

The model can be used to compare the performances of a one-impact and a two-impact armor. (The characteristics of the two different armors are not arbitrary but have been chosen to see where there might be an advantage to a two-impact armor.) The basic dimensions of both armors are the same: D = 100mm and $\delta = 0$. A cell in the one-impact armor is able to defeat the threat at muzzle velocity (900 m/s), and a cell in the two-impact armor is capable of defeating the same threat at 850 m/s. However, at an impact velocity of 800 m/s and below, a cell in the two-impact armor can defeat two impacts. Assume the same velocity retardation as given in equation 8, let n = 7, and take σ = 1 mrad. The value of P as a function of range for these two armors is shown in Figure 3.

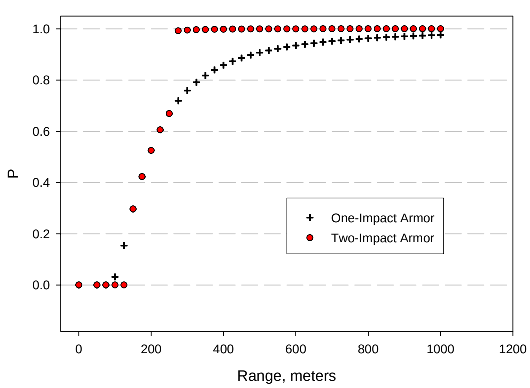

Figure 3. Performance comparison for one- and two-impact armors and n = 7 as a function of range

At short range (less than 50 meters) neither armor is capable of defeating seven impacts because the impacted area contains only a few cells. From 50 meters to 150 meters the one-impact armor outperforms the two-impact armor by a very small margin. From 150 meters to 250 meters the armor performances are the same. (The threat projectile velocity has decreased to the point where the cells in the two-impact armor can now defeat it.) For ranges 250 meters and above, the two-impact armor performance exceeds that of the one-impact armor. The difference decreases as the range increases.

The examples presented here of the model calculations are illustrative only and serve to show how the model can be exercised. Even so, there are two general observations that can be made concerning patterned armor performance. First, the model indicates the importance of eliminating any areas of weak performance near the edges of the cell. Second, the ability for a cell to defeat the threat at muzzle velocity is reduced at close range where the probability is high that a single cell will have several impacts.

2. COMPUTER MODEL

Two deficiencies are apparent in the analytic model. First, it is assumed that there is a large number of cells within the impact area A, and second, the impacts land in a uniform random pattern over the area A. In order to address these deficiencies, a computer model was written in software readily available at both the Army Research Laboratory and the United States Military Academy (Mathematica™). The model allowed for a Gaussian distribution of impacts falling in an area that was characterized by the standard deviation from the aim point. The text of the code has been documented (de Rosset and Sturdivant, 2005) and is not reproduced here.

The code first sets the armor array. The input parameters are the armor height and width, along with the number of cells along the horizontal and vertical directions. This input could be the actual dimensions for an armored vehicle, enabling the code to be applied to actual survivability problems. Currently, the cells are arranged in checkerboard array. They could also be arranged so that the cells were offset by half an edge length.

The aim point is designated by the values of two coordinates. In addition, the standard deviation from the aim point in both coordinate directions is specified. Note that if the aim point is near the edge of the patterned armor array, then there exists the possibility that one or more of the rounds in the burst of fire could totally miss the array. The code assigns an impact point for each of n shots in the burst based on a Gaussian distribution around the aim point. For each set of

n shots the code keeps a record of all the cells that have been perforated more than once, indicating a target defeat. The calculation is repeated, and after a large number of iterations an average (expressed as a per cent) is taken. An estimate of the probability that the armor is not defeated is 1 minus this average. The larger the number of iterations that are made, the closer this number will be to the true probability that the armor defeats the first *n* shots. This estimate of the probability will be referred to as *P*. It was found that after 3000 iterations, the average generally approached a single value. For this report, the number of iterations was 5000.

The code allows for a zone of low performance near the edge of the cell. The input specifies the value of δ to be used. It also allows for the individual cells to withstand more than one impact.

A comparison can be made between the analytic model and the computer model for input conditions that are applicable to the analytic model. Assume that D = 100 mm, A = 2 square meters, and $\delta = 0$. The value of P_n will be that shown in Figure 1. These input parameters give the standard deviation as 0.479 m (see equations 6 and 7). The computer model has a finite number of cells in the array. For this particular case, an array of 20 by 20 cells, each 100 mm on an edge, was used. The aim point was in the center of the array. This allowed for a 2-μ spread of shot locations on the array. While this does not capture all the impacts, a reasonable estimate of the probability that the array will survive *n* shots can be made. The code was run three times for each value of *n*, and an average of the three runs for each *n* is shown in Figure 4, along with the analytic model results.

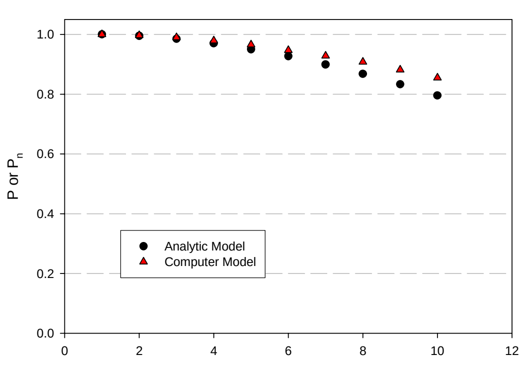

Figure 4. Comparison of analytic and computer model results

The results for the two approaches begin to diverge slightly for *n* greater than 5. The lower values calculated with the analytic model may be due to the fact that when a cell on the boundary of the impacted area is hit, it removes (mathematically) the entire cell from the area *A* even though only a portion of the cell may actually be contained in *A*. The computer model may over-estimate the probability of defeating *n* impacts because there is the possibility that not all of the impacts hit the armor array. In any event, the close agreement provides mutual support for the validity of both approaches.

The computer code is especially suited for calculating the effects of a small number of shots in a burst fired from an automatic weapon at close range. For those cases in which the weapon is hand-held, the standard deviation can be rather large. (This is in contrast to the case in which the weapon is fired from a fixed mount and its dispersion is better defined.) In these cases, the chosen value of σ is somewhat arbitrary, although a conservative approach to armor design would consider *sd* values that are comparable to those associated with fixed-mount systems.

An example calculation can be made to determine probability of projectile defeat for small numbers of projectiles in a burst at short range. Consider a 5 by 5 array of cells each with an edge length of 100mm. Assume that δ = 0 and that the range is 50 meters. The aim point is the center of the armor array. Let σ = 3 mrad. Figure 5 shows the average probability, *P*, that the armor will defeat the first *n* projectiles.

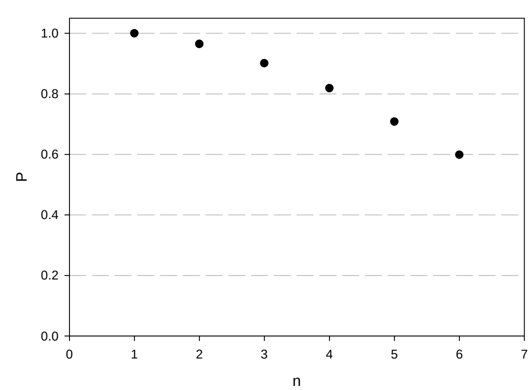

Figure 5. Computer code results for short range and low *n*

In this example, a value of 3 mrad for σ was chosen arbitrarily. For a given range, its value will be an important factor in the resulting value of *P*. This is shown in Figure 6, where *P* is calculated for several values of σ with n = 4 and all other input parameters the same as in the preceding example. The figure shows that the average probability of defeating 4 impacts rises steeply for small values of σ but then tends to level off for values of σ above 3 mrad..

For weapons with very low dispersion the calculated value of *P* will depend strongly on the aim point. Figure 7 shows three possible aim points on a patterned armor as referenced to the cell location: center, edge, and corner. (The edge aim location is specifically on a seam half-way between the cells' corners.) The value of *P* for each of these aim points is shown as a function of *n* in Figure 8. For this example, σ = 0.25 mrad. The corner aim point has the highest values of *P* for a given value of *n* because the impacts are spread, for the most part, over four cells. Similarly, the impacts for the edge aim point are generally spread over two cells. The dispersion is so low that the impacts for the center aim point fall primarily within one cell. For σ greater than 1 mrad, very little effect of aim point is observed.

.

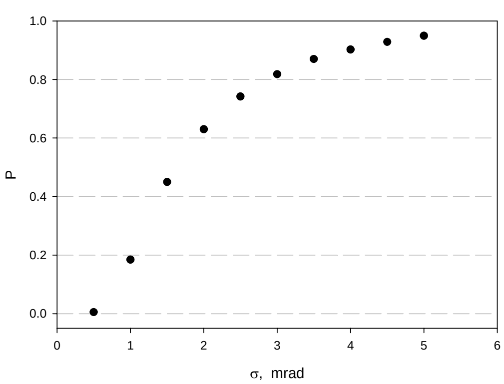

Figure 6. Calculated average probability of defeating 4 impacts as a function of the dispersion

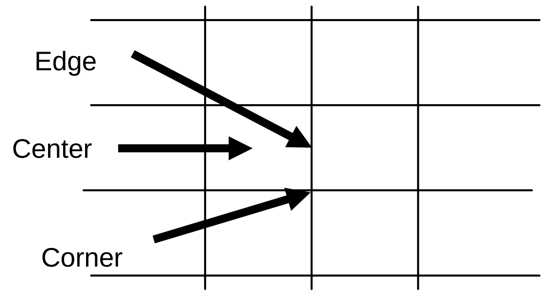

Figure 7. Aim point designations on a patterned armor

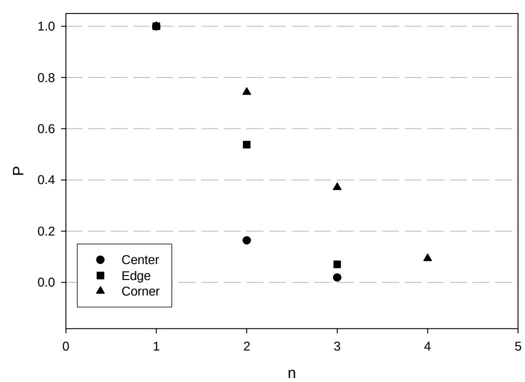

Figure 8. *P* as a function of *n* for various aim points of a weapon with low dispersion

CONCLUSIONS

Two approaches to modeling multiple impacts on patterned armor have been developed to aid the armor designer in assessing performance of different armor concepts. The first is an analytic model that can be applied to situations where there is a large number of cells in the patterned armor falling within the dispersion zone of the weapon. The model shows the importance of the individual cell to defeat an impact over its entire area. The second approach uses a computer code to model a normal distribution of impacts over the dispersion zone and is specifically developed to address small numbers of impacts. The code calculations show that weapon dispersion is an important parameter for determining the performance of patterned armor.

REFERENCES

de Rosset, W.S. and Wald, J.K., 2003: Analysis of Multiple-Hit Criterion for Ceramic Armor, ARL Technical Report ARL-TR-2861

de Rosset, W.S., 2005: Patterned armor performance evaluation, *Int. J. Imp. Eng.*, **31,** 1223-1234.

de Rosset, W.S. and Sturdivant, R.X., 2005: Investigation of Multiple Hits on Light Armor, ARL Technical Report ARL-TR-3623